Evaluate the following integrals. **SCORE: _____ / 50 PTS**

[a] $\displaystyle\int \frac{80-x^4}{x^3+4x^2+20x}\,dx$

$$\begin{array}{r l}
 & -x+4 \\
x^3+4x^2+20x \,\big)\!\! & -x^4 \qquad\qquad +80 \\
 & -x^4-4x^3-20x^2 \\
 & 4x^3+20x^2 \\
 & 4x^3+16x^2+80x \\
 & 4x^2-80x+80
\end{array}$$

$$\frac{4x^2-80x+80}{x(x^2+4x+20)} = \frac{A}{x} + \frac{B(2x+4)+C(4)}{(x+2)^2+16}$$

$$4x^2-80x+80 = A[(x+2)^2+16] + Bx(2x+4) + C(4x)$$

$x=0$: $80 = 20A \to A=4$

$x=-2$: $16+160+80 = 4(16)-8C$

$256 = 64-8C$

$C=-24$

COEF OF x^2: $4 = 4+2B \to B=0$ 4

CHECK $x=2$: $\frac{80-16}{8+16+40} \stackrel{?}{=} -2+4+\frac{4}{2}-\frac{24(4)}{16+16}$

$\frac{64}{64} \stackrel{?}{=} -2+4+2-\frac{96}{32}$

$1 = -2+4+2-3$ ✓

$$\int\left(-x+4+\frac{4}{x}-\frac{24(4)}{(x+2)^2+16}\right)dx$$

4 4

$$= -\frac{1}{2}x^2+4x+4\ln|x|-24\tan^{-1}\frac{x+2}{4}+C$$

2 2 4

3 ALL OTHER ITEMS

[b] $\displaystyle\int e^{-2x}\sin 6x\,dx$

u	dv
$\sin 6x$ $+$	e^{-2x}
$6\cos 6x$ $-$	$-\frac{1}{2}e^{-2x}$
$-36\sin 6x$ $+$	$\frac{1}{4}e^{-2x}$

$$\int e^{-2x}\sin 6x\,dx = -\frac{1}{2}e^{-2x}\sin 6x - \frac{3}{2}e^{-2x}\cos 6x - 9\int e^{-2x}\sin 6x\,dx$$

$$10\int e^{-2x}\sin 6x\,dx = -\frac{1}{2}e^{-2x}\sin 6x - \frac{3}{2}e^{-2x}\cos 6x$$

$$\int e^{-2x}\sin 6x\,dx = -\frac{1}{20}e^{-2x}\sin 6x - \frac{3}{20}e^{-2x}\cos 6x + C$$

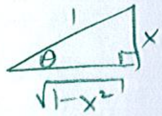
3 EACH

Evaluate the following integrals. **SCORE: _____ / 55 PTS**

[a] $\int (\arcsin x)^2\,dx$

4 $\underline{\theta = \arcsin x}$

$x = \sin\theta$ $\longrightarrow$



$dx = \cos\theta\,d\theta$

4 $\underline{\int \theta^2 \cos\theta\,d\theta}$

u	dv
θ^2	$\cos\theta$
2θ	$\sin\theta$
2	$-\cos\theta$
0	$-\sin\theta$

8 $= \theta^2 \sin\theta + 2\theta\cos\theta - 2\sin\theta + C$

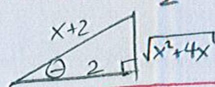
4 $= x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$

[b] $\int \frac{x^2}{\sqrt{x^2+4x}}\,dx = \int \frac{x^2}{\sqrt{(x+2)^2-4}}\,dx$

$(x+2)^2 - 4 = 4\sec^2\theta - 4 = 4\tan^2\theta$

$\sec\theta = \frac{x+2}{2} \longleftarrow \underline{x = 2\sec\theta - 2}$

$dx = 2\sec\theta\tan\theta\,d\theta$



$\int \frac{(2\sec\theta - 2)^2}{\cancel{2}\cancel{\tan\theta}}\, \cancel{2}\sec\theta\, \cancel{\tan\theta}\,d\theta$

$= 4\int (\sec^3\theta - 2\sec^2\theta + \sec\theta)\,d\theta$

$= 4\Big(\underbrace{\tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\ln|\sec\theta + \tan\theta|}_{4} \underbrace{- 2\tan\theta}_{2} + \underbrace{\ln|\sec\theta + \tan\theta|}_{2}\Big) + C$

$= 2\sec\theta\tan\theta + 6\ln|\sec\theta + \tan\theta| - 8\tan\theta + C$

$= 2\left(\frac{x+2}{2}\right)\left(\frac{\sqrt{x^2+4x}}{2}\right) + 6\ln\left|\frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2}\right| - 8\left(\frac{\sqrt{x^2+4x}}{2}\right) + C$

$= \frac{1}{2}(x+2)\sqrt{x^2+4x} + 6\ln\left|x+2+\sqrt{x^2+4x}\right| - 4\sqrt{x^2+4x} + C$

$= \frac{x-6}{2}\sqrt{x^2+4x} + 6\ln\left|x+2+\sqrt{x^2+4x}\right| + C$

3 ALL OTHER ITEMS

Determine if $\int_1^\infty \frac{2+\cos x}{e^x}\,dx$ converges or diverges. Justify your answer properly. **SCORE: _____ / 20 PTS**

$$0 \le \frac{2+\cos x}{e^x} \le \frac{3}{e^x} = 3\left(\frac{1}{e}\right)^x$$

(3) (8)

(3) $3\int_1^\infty \left(\frac{1}{e}\right)^x dx$ CONVERGES $\left(0 < b = \frac{1}{e} < 1\right)$ (3)

SO $\int_1^\infty \frac{2+\cos x}{e^x}\,dx$ CONVERGES

(3)

Find $\int_{-\pi}^{\pi} \tan^4 x \sec^6 x\,dx$. **SCORE: _____ / 25 PTS**

$$= \int_{-\pi}^{-\frac{\pi}{2}} \tan^4 x \sec^6 x\,dx + \int_{-\frac{\pi}{2}}^{0} \tan^4 x \sec^6 x\,dx + \int_0^{\frac{\pi}{2}} \tan^4 x \sec^6 x\,dx + \int_{\frac{\pi}{2}}^{\pi} \tan^4 x \sec^6 x\,dx$$

(3)

$$\int_0^{\frac{\pi}{2}} \tan^4 x \sec^6 x\,dx$$

(3) $= \lim\limits_{N \to \frac{\pi}{2}^-} \int_0^N \tan^4 x \sec^6 x\,dx$

$= \lim\limits_{N \to \frac{\pi}{2}^-} \left(\frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x\right)\Big|_0^N$

(3) $= \lim\limits_{N \to \frac{\pi}{2}^-} \left(\frac{1}{9}\tan^9 N + \frac{2}{7}\tan^7 N + \frac{1}{5}\tan^5 N\right)$

$\downarrow \infty \quad \downarrow \infty \quad \downarrow \infty$

(3) $= \infty$

(3) DIVERGES

$u = \tan x$ $(2\frac{1}{2})$

$du = \sec^2 x\,dx$

$\int \tan^4 x \sec^4 x \sec^2 x\,dx$

$= \int u^4(u^2+1)^2\,du$ $(2\frac{1}{2})$

$= \int (u^8 + 2u^6 + u^4)\,du$

$= \frac{1}{9}u^9 + \frac{2}{7}u^7 + \frac{1}{5}u^5$ $(2\frac{1}{2})$

$= \frac{1}{9}\tan^9 x + \frac{2}{7}\tan^7 x + \frac{1}{5}\tan^5 x$

$(2\frac{1}{2})$